Final: Sat, March 10, 5:00-7:50pm,
Final Room is based on quiz section
For BC/BD, AC/AD: PAA A102
For $A A / A B, B B$ PAA A118
For BA:
PAA A110

## Entry Task: Course Evaluation

Get out computer/phone, fill out the evaluation (first 10 min of class):

## 112A Course Eval:

## uw.iasystem.org/survey/185700

 112B Course Eval:uw.iasystem.org/survey/185699

If you are done get out old final questions and work on them until we start, lecture.

## Evaluation Notes

- This eval. is for me and the lecture/class (your TA will have a different eval. for quiz section).
- I will not see the results until next quarter (I will never see your name)
- The comments only go to me.

Course best described as...:
"In your major" means you're a math major. For the vast majority of you, this course is a "core/distribution requirement".

## EXAM 1 MATERIAL

Functional Notation,
Definition of Deriv,
Deriv Rules, Tangent line equation,
What does the deriv. represent?

## EXAM 2 MATERIAL

More Deriv Rules,
Critical Points, Local Max/min
Absolute max/min, inc/dec, Concave up/down, Inflection points.
Integrals and integral notation,
Evaluating integrals,
What does the integral represent?

## NEW MATERIAL

More integral applications
Multivariable functions,
Partial Derivatives, Critical Points,
What do partial deriv. represent?
$\qquad$
$\qquad$
Student ID \#: $\qquad$

# Math 112 -- Winter 2016 <br> Final Exam 

## HONOR STATEMENT:

"I affirm that my work upholds the highest standards of honesty and academic integrity at the University of Washington, and that I have neither given nor received any unauthorized assistance on this exam."

SIGNATURE: $\qquad$

## INSTRUCTIONS:

- When the exam starts, verify that your exam contains $\mathbf{9}$ pages (including this cover page).
- Please turn your cell phone OFF and put it away for the duration of the exam.
- Unless specifically instructed otherwise, you must show all your work in order to get full credit. The correct answer with incorrect or missing work may result in little or no credit.
- On problems in which you use a graph, show your work by clearly drawing \& labeling any lines and points you use.
- If you use a guess-and-check method when an algebraic method is available, you will not receive full credit.
- You may round your final answers to two decimal digits. Don't round any values prior to the final answer.
- You are allowed to use a calculator, a ruler, and one sheet of notes. You have 2:50 hours for this exam.


## GOOD LUCK!

| Problem 1 | 16 |  |
| :---: | :---: | :---: |
| Problem 2 | 6 |  |
| Problem 3 | 10 |  |
| Problem 4 | 8 |  |
| Problem 5 | 12 |  |
| Problem 6 | 12 |  |
| Problem 7 | 10 |  |
| Problem 8 | 14 |  |
| Problem 9 | 12 |  |
| Total: | $\mathbf{1 0 0}$ |  |

1 (16 pts) Compute the indicated derivatives. DO NOT SIMPLIFY. Box your final answer.
a) $\quad f(t)=\sqrt{\ln \left(t^{2}-3 t\right)+7}$

$$
f^{\prime}(t)=
$$

b) $\quad u=\frac{e^{x} \ln x}{x^{2}+\frac{1}{x}-7}$

$$
\frac{d u}{d x}=
$$

c) $\quad z=2 e^{y} x+\frac{y}{x}+\ln \left(x y^{2}\right)+x$

$$
\frac{\partial z}{\partial x}=
$$

$$
\frac{\partial z}{\partial y}=
$$

2 (6 pts) Suppose we do not have a formula for a certain function $f(x)$, but we know that:

$$
f(m+h)-f(m)=\frac{12 h}{(2+m+h)(5+m)}
$$

Compute $f^{\prime}(3)$. Show all steps clearly.

ANSWER: $f^{\prime}(3)=$
3 (10 pts) Compute each of the following integrals. SIMPLIFY and box your final answers.
a) $\int \frac{3}{x^{2}}-2 e^{2 x}+\frac{7 x^{2}+3}{x} d x$
b) $\int_{9}^{25} \frac{3}{\sqrt{t}}+2 d t$

4 (8 pts) The demand and supply functions for a product are:

$$
\begin{array}{lc}
\text { demand: } & p=\frac{77}{x+1} \\
\text { supply: } & p=2+0.5 x
\end{array}
$$

where $p$ is the price per unit, in dollars, and $x$ is the number of units. Compute the consumers surplus under pure competition.

ANSWER: Consumers Surplus = \$ $\qquad$
(You may round your final answer to the nearest two decimal digits)

5 (12 pts) Two bicyclists, Anne and Bob, are next to each other at time $t=0$, and travel along the same straight road. Their respective speeds at $t$ hours are given by the functions:

$$
\begin{array}{ll}
\text { Biker Anne's speed: } & \boldsymbol{a}(\boldsymbol{t})=\mathbf{3} \boldsymbol{t}^{\mathbf{2}}-\mathbf{1 0 t}+\mathbf{1 6} \text { miles/hour } \\
\text { Biker Bob's speed: } & \boldsymbol{b}(\boldsymbol{t})=\mathbf{2 t}+\mathbf{1 0} \quad \text { miles/hour }
\end{array}
$$

a) At what time during the first 1.5 hours are the two bikers farthest apart?

Answer: at $t=$ $\qquad$ hours.
b) Which biker is ahead after 1 hour, and by how much? Show work.

Answer: Biker $\qquad$ is ahead by $\qquad$ miles.
c) Recall that the instantaneous speed for Biker Bob is given by the linear function: $b(t)=2 t+10$. Compute the average speed of Biker Bob over the time interval from $t=1$ to $t=2.5$ hours.
$\qquad$ miles per hour.

6 (12 pts) The marginal revenue and marginal cost at $q$ hundred Things are given by the graphs below.


You also know that your fixed costs are 2 hundred dollars.
a) Estimate your Total Cost for producing 300 Things. Show your work.

Answer: TC (3) $\approx$ $\qquad$ hundred dollars
b) Estimate the minimal profit (maximal loss), and the quantity at which it occurs. Show work.

Answer: Min Profit / Max Loss $\approx$ $\qquad$ hundred dollars, at $q \approx$ $\qquad$ hundred Things
c) Estimate the change in revenue from $q=3$ to $q=4$ hundred Things. Show work.

Answer: $\qquad$ hundred dollars
d) Does your profit increase or decrease if you produce and sell the $301^{\text {st }}$ Thing? By approximately how much?
$\qquad$ dollars

7 (10 pts) The following is the graph of a function $f(t)$.


Let $\boldsymbol{A}(\boldsymbol{m})=\int_{\mathbf{0}}^{\boldsymbol{m}} \boldsymbol{f}(\boldsymbol{t}) \boldsymbol{d} \boldsymbol{t}$ be the accumulated graph of $f(t)$. Answer the following questions. Read each question carefully!
a) For each part below, circle the correct answer. No need to justify.
i. The value of $f(5)$ is POSITIVE, NEGATIVE, or ZERO
ii. The value of $f^{\prime}(5)$ is POSITIVE, NEGATIVE, or ZERO
iii. The value of $f^{\prime \prime}(5)$ is POSITIVE, NEGATIVE, or ZERO
iv. The value of $A(7)$ is POSITIVE, NEGATIVE, or ZERO
v. The value of $A^{\prime}(7)$ is POSITIVE, NEGATIVE, or ZERO
b) Find the longest interval during which the derivative $f^{\prime}(t)$ is decreasing.

Answer: from $t=$ $\qquad$ to $t=$ $\qquad$
c) Estimate $A^{\prime}(\mathbf{9})$.

Answer: $A^{\prime}(9) \approx$ $\qquad$
d) $f(t)$ has inflection points at $x=$ $\qquad$ (list all, no need to justify)
e) The local minima of $\boldsymbol{A}(\boldsymbol{m})$ are at $m=$ $\qquad$ (list all, no need to justify)

8 (14 Points) You produce and sell flat-screen TV's and Blu-ray Players.
(a) ( 2 pts ) Suppose you sell each TV for $\$ 2000$ and each Player for $\$ 500$. Give a formula for the total revenue $R(x, y)$, in dollars, which results from selling $x$ TV's and $y$ Players.

ANSWER: $R(x, y)=$
(b) Suppose your profit from selling $x$ TV's and $y$ Players is given by the function:

$$
P(x, y)=0.1 x^{2}+0.1 y^{2}-0.6 x y+300 x+100 y-1000
$$

i. (2 pts) Compute the two partial derivatives of your profit function.

$$
\begin{aligned}
& P_{x}(x, y)= \\
& P_{y}(x, y)= \\
& \hline
\end{aligned}
$$

ii. (6 pts) Find all candidates $(x, y)$ for local minima or maxima of the profit $P(x, y)$.

Answer: $(x, y)=$ $\qquad$
iii. (4 pts) Suppose you've produced and sold 300 TV's and 250 Players. Use a partial derivative to estimate the increase in your profit if you sell one more TV. Show your work, clearly.
$\qquad$

9 (12 pts) The Demand Curve for selling Items has the formula:

$$
p=1-0.2 \sqrt{q},
$$

where the quantity $q$ is in hundreds of Items and the price $p$ is in dollars per Item.
The total cost (in hundreds of dollars) to produce $q$ hundred Items is given by the formula:

$$
T C(q)=0.01 q+0.5
$$

Let $\boldsymbol{P}(\boldsymbol{q})$ denote the profit (in hundreds of dollars) you earn by producing and selling $q$ hundred Items.
a) Determine the formula for the profit $P(q)$, as an expression in $q$. Simplify your answer.

ANSWER: $P(q)=$
b) Compute the critical number(s) of the profit.

ANSWER: $q=$ $\qquad$ hundred Items
c) Use the Second Derivative Test to determine whether each critical number you found above gives a local maximum or a local minimum for the profit function, $P(q)$. Show work clearly, and box your answer(s).

